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ON THE DIOPHANTINE EQUATION $p^x + q^y = z^2$ By

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ABSTRACT:
In this paper, Diophantine equation $\boldsymbol{p^x + q^y = z^2}$ has been discussed for some values of prime numbers p_1 and p_2. Positive integral solutions of the above Diophantine equation have been obtained.

1 INTRODUCTION:
Catalan, E. (1844) conjectured that $(3,2,2,3)$ is a unique solution (a,b,x,y) for the Diophantine equation $a^x - b^y = 1$ where a, b, x and y are with $\min(a,b,x,y) > 1$. This conjecture was proved by **Mihailescu** (2004).
Acu (2007) showed that $(3,0,3)$ and $(2,1,3)$ are the only solutions (x,y,z) for the Diophantine equation $2^x + 5^y = z^2$ where x, y and z are non-negative integers. **Suvarnamani et. al** (2011) showed that two Diophantine equations $4^x + 7^y = z^2$ and $4^x + 11^y = z^2$ have no non-negative integer solution. **Suvarnamani** (2011) obtained some non-negative integer solution of the Diophantine equation $2^x + p^y = z^2$ where p is a prime number. **Chotehaisthit** (2012) obtained all non-negative solution of the Diophantine equation $4^x + p^y = z^2$ where p is a prime number. **Banyat Sroysang** (2012) showed that the Diophantine equation $3^x + 5^y = z^2$ has a unique solution. **Banyat Sroysang** (2012) obtained the solution of Diophantine equations $8^x + 19^y = z^2$ and $31^x + 32^y = z^2$. He showed that the first Diophantine equation has a solution (1, 0, 3) and the second Diophantine equation has no non-negative integer solution. **Catalan's** conjecture was used to obtain the solutions of the above Diophantine equations. He proposed open problems.

In this paper, the Diophantine equation $\boldsymbol{p^x + q^y = z^2}$ where p and q are prime numbers has been discussed for its non-negative integer solutions. Attempt has been made to work on the above mentioned open problems.

2. RESULTS:
Problem 1: Solution of the Diophantine equation $2^x + 7^y = z^2$ are given by $(x,y,z) = (3,0,2)$ and $(1,1,2)$.
Problem 2: Solution of the Diophantine equation $2^x + 17^y = z^2$ are given by $(x,y,z) = (3,0,3)$, $(5,1,7)$, $(3,1,5)$, $(6,1,9)$ and $(7,3,71)$.
Solution: It can be verified that $(x,y,z) = (3,0,3)$, $(5,1,7)$, $(3,1,5)$, $(6,1,9)$ and $(7,3,71)$ satisfy the given Diophantine equation.

Problem 3: Solution of the Diophantine equation $2^x + 31^y = z^2$ are given by $(x,y,z) = (3,0,3)$ and $(7,2,33)$.
Solution: It can be verified that $(x,y,z) = (3,0,3)$ and $(7,2,33)$ satisfy the given Diophantine equation.

Problem 4: Solution of the Diophantine equation $2^x + 41^y = z^2$ is given by $(x,y,z) = (3,1,7)$.
Solution: It can be verified that $(x,y,z) = (3,1,7)$ satisfies the given Diophantine equation.

Problem 5: Solution of the Diophantine equation $2^x + 47^y = z^2$ are given by $(x, y, z) = (1,1,2), (3,1,2)$ and $(5,1,2)$.
Solution: It can be verified that $(x, y, z) = (1,1,2), (3,1,2)$ and $(5,1,2)$ satisfy the given Diophantine equation.

Problem 6: Solution of the Diophantine equation $2^x + 73^y = z^2$ is given by $(x, y, z) = (3,1,4)$.
Solution: It can be verified that $(x, y, z) = (3,1,4)$ satisfies the given Diophantine equation.

Problem 7: Solution of the Diophantine equation $2^x + 79^y = z^2$ is given by $(x, y, z) = (1,1,4)$.
Solution: It can be verified that $(x, y, z) = (1,1,4)$ satisfies the given Diophantine equation.

Problem 8: Solution of the Diophantine equation $5^x + 7^y = z^2$ has no integral solution.
Problem 9: Solution of the Diophantine equation $11^x + 13^y = z^2$ has no integral solution.
Problem 10: Solution of the Diophantine equation $17^x + 19^y = z^2$ is given by $(x, y, z) = (1,1,6)$.
Solution: It can be verified that $(x, y, z) = (1,1,6)$ satisfies the given Diophantine equation.

Problem 11: Solution of the Diophantine equation $71^x + 73^y = z^2$is given by $(x, y, z) = (1,1,12)$.
Solution: It can be verified that $(x, y, z) = (1,1,12)$ satisfies the given Diophantine equation.

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